

A Summary of the Panel Discussion on Sunday, May 9, 2004 at the Cornell Topology Festival

May 25, 2004

During the past eighteen months, G. Perelman's announcement and description of his proof of the Poincaré Conjecture and of Thurston's Geometrization Conjecture, via the Ricci-flow program of R. Hamilton, has greatly excited the mathematical community, which awaits a final vetting of the asserted results. The organizers of the Cornell Topology Festival felt that this was an appropriate time to focus on the area of three-dimensional manifolds, both with an eye to other current important results in three-manifold theory and with an eye to what the future may hold. Accordingly, about one half of the talks at the Festival focused on this area. Also, in addition to the eleven research talks, there were two survey talks, or workshops, intended to serve as background on three-manifolds for non-experts. The Festival also conducted a panel discussion again this year, organized similarly to last year's successful discussion, with each speaker being asked to prepare a description of some recent mathematics which had caught his or her interest, outside of the speakers own work. Karen Vogtmann, one of the Cornell organizers of the Festival, served again as moderator for the discussion. She asked speakers to limit their remarks to one blackboard's length, a request honored by some.

In this summary of the panel discussion, a panelist's name is indicated in bold-face type, whereas names of mathematicians whose work receives significant mention are indicated in italic. Panelist comments are described in the order presented at the session.

John Etnyre (University of Pennsylvania) was impressed by a result of *Gay* and *Kirby* on the important problem of better understanding symplectic 4-manifolds. Their result gives an explicit construction of closed 2-forms on smooth 4-manifolds with $b_2^+ > 0$; the 2-forms are symplectic on the complement of a disjoint union of finitely many embedded circles.

Yuli Rudyak (University of Florida) spoke of results from independent work of *Iwase* and *Stanley*. They were able to provide counterexamples to conjectures of *Ganea* on the Lyusternik-Shnirelman category of a space. Recall that the category of a space is the minimal cardinality of an open contractible cover. The counterexamples answered negatively the questions of whether: (a) the category of a manifold M is one less than the category of $M \times S^n$; (b) the category of M minus a point is one less than the category of M .

Peter Kronheimer (Harvard University) shared a result of *Fintushel* and *Stern*. The basic idea is to create invariants that discern differences in submanifolds when embedded into a larger

manifold. Fintushel and Stern show that certain invariants survive when Lagrangian tori are sewed into a class of symplectic 4-manifolds.

Richard Schwartz (University of Maryland) recently enjoyed results of *Mirzakhani* about simple closed curves on closed hyperbolic surfaces. Among other results, she showed that the number of curves with length less than n is asymptotic to $n^{(6g-6)}$ where g is the genus of the surface.

Zoltan Szabo (Princeton University) offered work of *Rasmussen* who gave a new combinatorial proof to the *Milnor* unknotting conjecture, which was proved by *Kronheimer* and *Mrowka* in 1992. The Milnor conjecture states that the minimum genus of a surface embedded in a 4-ball which bounds a (p, q) -torus knot is equal to $(p-1)(q-1)/2$.

Nathan Dunfield (California Institute of Technology) described the *Garoufalidis* and *Le* theorem that the colored Jones function is q -holonomic. From the abstract of their paper: A function of several variables is called holonomic if it satisfies a maximally overdetermined system of linear differential equations with polynomial coefficients. The colored Jones function is an invariant for knots in the 3-sphere.

Dennis Sullivan (City University of New York) provided a sample from the contents of a book by *Y. Manin* on Frobenius manifolds, quantum cohomology, and moduli spaces.

Jeffrey Brock (University of Texas, Austin and Brown University) talked to the audience about a recent theorem of *Juan Souto*. The result is that any sufficiently short closed geodesic in a hyperbolic handlebody must be isotopic into the boundary of the handlebody as long as the curve intersects each essential disc at least twice and each essential annulus at least once. This result is reminiscent of *Otal's* theorem that any sufficiently short geodesic in a hyperbolic manifold that is diffeomorphic to the product of a surface and a real line, must be unknotted.

Ian Agol (University of Illinois, Chicago) explained some recent work of *Angenent* and *Knopf* who provide a version of *Sturmian's* theorem applied to Ricci flow. Recall that Sturmian's theorem shows that real valued functions on a cylinder satisfying the heat equation are analytic.

William Thurston (Cornell University), noting that the time was getting late, postponed his remarks till his scheduled talk the next day, which was intriguingly entitled, "What Next?" In his talk, he spoke on the current state of the field of 3-manifolds. With the ending lamination and the tameness conjectures resolved, and a confirmation of the Geometrization Conjecture on the horizon, much of the program originally proposed by Thurston is at a conclusion. This does not mean the field has come to an end. For example, the classification of surfaces has existed for a long time, but surfaces are still widely studied. Thurston also remarked that we shouldn't end our attempts to understand our new-found theorems on an even deeper level. In particular, the theory of 3-manifolds could benefit from a better overall exposition that would be easier to export to other fields.