Panel Discussion at the Cornell Topology Festival

May 9,2005

This year the Topology Festival had as its area of concentration symplectic geometry/topology, with about one third of the talks, and both workshops, devoted to this. As has become customary, the speakers were also featured in a two-hour panel discussion on Sunday, May 9, in which they presented some recent results that interested them but were not part of their own work. **Peter Kahn**, one of the Cornell organizers, acted as moderator, i.e., timekeeper. He requested that the speakers keep remarks limited to approximately "five minutes or one blackboard, whichever comes first."

In this summary of the panel discussion, a panelist's name is indicated in bold-face, whereas the names of mathematicians whose work receives significant mention are indicated in italics.

Etienne Ghys of ENS-Lyon described work of *Julien Marche* of CRAS on the space $\mathcal{K}$ of all knots in 3-space, which is endowed with the metric that counts the minimum number of crossings and uncrossings needed to transform one knot into another. He is interested in the "rough geometry" of $\mathcal{K}$, and he cited a result of *Giambaudo* to the effect that $\mathbb{Z}^n$ admits a quasi-isometric embedding into $\mathcal{K}$. He conjectured that one could do the same for an infinite trivalent tree. In general, he presented a family of open questions: Take your favorite metric space X. Does X admit a quasi-isometric embedding into $\mathcal{K}$? Does X give a counterexample to this? Now replace $\mathcal{K}$ by $\mathcal{M}_3$, the set of closed 3-manifolds with distance given by the minimum number of Morse surgeries required to go from one manifold to another manifold. Ask the same questions for $\mathcal{M}_3$. Here, an answer is not even known for $\mathbb{Z}^n$ or an infinite trivalent tree.

Augustin Banyaga of Pennsylvania State University discussed the renowned Flux Conjecture, a version of which first appeared in his thesis in the 1970's. The Flux Conjecture may be formulated in terms of the subgroup $Ham(M) \subseteq Symp(M)$, consisting of those symplectomorphisms that are Hamiltonian. The conjecture is that this subgroup is closed. Another formulation considers a natural homomorphism $\pi_1(Symp(M)) \to H^1(M;\mathbb{R})$, defined by integrating the symplectic form of M over certain singular surfaces in M, and conjectures that its image is discrete. Banyaga then described the affirmative solution to this conjecture by *K. Ono*, which makes use of the Novikov-Floer homology of M.

Brendan Owens of Cornell University described work of *Tim Perutz* that makes use of the Heegaard-Floer theory of Ozvath and Szabo.

Thomas Delzant of IRMA Strasbourg described work of *Petronio Perva* involving the complexity of 3-manifolds and Milnor groups. Given a closed 3-manifold M, one can define its complexity $c(M)$ to be the minimum number of triangles in a triangulation of M. Given a group presentation $\mathcal{P} =< a_1, \ldots, a_n : R_1, \ldots, R_k >$, one can define its complexity $c(\mathcal{P})$ to be the sum of the reduced word lengths of the R_i. For any finitely-presented group G, set $c(G) = min(c(\mathcal{P}))$, where $\mathcal{P}$ ranges over all finite presentations of G. Question: What is the relationship between $c(M)$ and $c(\pi_1(M))$? When M is a lens space with fundamental group $\mathbb{Z}/n\mathbb{Z}$, we know that the two are asymptotically equal to $\ln(n)$.

John Morgan of Columbia University discussed combinatorial 3-manifold thoery: in particular, Khovanov homology, which is defined purely combinatorially. Given the double cover X of the complement of a knot, there exists a spectral sequence with E^2-term the Khovanov homology of X, which converges to the Heegard-Floer homology of a Heegaard splitting of X. Question: In general, can one get the manifold-theoretic results coming from gauge theory via combinatorial means?

Yann Ollivier of ENS-Lyon spoke about work of *Karl-Theodor Sturm and Marc-Konstantin von Renesse* (and expanded by John Lott and Cedric Villani). The basic problem takes place in a Riemannian manifold M, in which one wishes to " move heaps of sand 'cheaply'". One unit of sand moved distance ℓ should have cost proportional to ℓ^2. Of course, a "sand" distribution refers to a probability measure on M. If the measure is transported through negative curvature, it shrinks; through positive curvature, it expands. Given a measure μ, define the entropy $Ent(\mu)$ to be the integral

$$\int \frac{d\mu}{dvol} \log \frac{d\mu}{dvol} dvol.$$

Then, the main result on this is the following: Let M be a Riemannian manifold, with metric g, and K any real constant. Then $Ric(M) \geq Kg \Leftrightarrow$ for all continuous transports of measures μ_t of μ_0 to μ_1, the function $t \mapsto Ent(\mu_t)$ is K-convex.

Denis Auroux of MIT spoke about fillability and open book decompositions. A result of *Eliashberg* assets that if a contact three-manifold is overstwisted, then it is not symplectically fillable (i.e., the boundary of a symplectic four-manifold with appropriate boundary data). The situation for tight contact structures is variable. Given any manifold M with boundary

∂M and self-diffeomorphism f of M, one can form, the data M, f is said to endow the mapping torus $M(f)$ with an open-book structure (*Winkelnkemper*). Giroux asserted that every contact three-manifold with boundary has an open-book structure. Question: Does this structure reveal whether the manifold is overtwisted, tight, symplectically fillable? Here are two theorems on this theme: 1) (*Giroux-Loi Piergallini*): A contact manifold with open book structure given by M, f is Stein-fillable if and only if f is a product of positive Dehn twists. 2) (*Honda-Kazez-Matic*): A contact three-manifold N is tight if and only if every open book decomposition M, f has f right-veering (which is detected by lifting to the universal cover and looking at the angle of rotation).

Shahar Mozes of Hebrew University discussed work of *Elon Lindenstrauss* concerning A-invariant, ergodic measures on arithmetic homogeneous spaces. He cited a conjecture of Margulies and Furstenberg on A-invariant probability measures on $SL_n(\mathbb{R})/SL_n(\mathbb{Z})$: namely, that they are algebraic.

Paul Biran of Tel Aviv University spoke about work of *Farber and Tabachnikov* on motion-planning in robotics theory. From a very general point of view, motion-planning concerns a space X in which one seeks an optimal path between two points. X represents the space of all possible locations of the robot in a certain physical environment. If PX denotes the space of paths in X, then we have the usual endpoints map $\pi : PX \to X \times X$. An F- solution to the motion planning problem is a continuous section $s : F \to PX|F$ of π, where F is a subset of $X \times X$. Problem: Does $X \times X$ admit a decomposition into finitely many disjoint sets F, each equipped with an F-solution, with each F an ENR? If so, what is the size S of the smallest such decomposition? Answers: When $X = S^n$, $S = 2$ when n is odd and 3 when n is even. When $X = \mathbb{R}P^n$, and $n \neq 1, 3, 7$, $S =$ the minimum k such that $\mathbb{R}P^n$ is immersible into $\mathbb{R}^{k-1}$. When X is a graph, $S = 1, 2, 3$, according as $b_1 = 0, 1$ or $b_1 \geq 2$. S may be compared to the Liusternik-Schnirelmann category as follows: $S \geq cat(X \times X/\Delta X) - 1$.