Ginzburg : Leaf-wise coisotropic intersections

Note Title 5/3/2008

Cornell

Lagrangian intersection

$L \subset W$ closed Lagrangian , $\varphi = \varphi_H \circlearrowright W$ Hamiltonian sympl.

$\Rightarrow \varphi(L) \cap L \neq \emptyset$ provided * "φ is small"

* $\varphi \overset{C^1}{\approx} id$ [Weinstein]
* $\varphi \overset{C^0}{\approx} id$ [Laudenbach - Sikorav] using generating function
* $\varphi \overset{C^{-1}}{\approx} id$ i.e. in Hofer metric [Floer, Oh, Chekanov] using Floer-homology

$$\|H\| = \int_0^1 (\max H_t - \min H_t)\, dt \quad \text{small}$$

• <u>C^0-case</u>

could assume φ is supported near L

• $W = \mathbb{R}^{2n}$, $\omega = d\lambda$ $\Rightarrow$ $[\lambda|_L] = 0$, Maslov $(L) \neq 0$

<u>Question</u> : Do these results have analogues for coisotropic submanifolds?

• Bolle (mid 1990s)

• V.G. (2007)

<u>TODAY</u> : further understanding based on work of Gurel

$M \subset W$ coisotropic

$\Rightarrow TM^\omega \subset TM$ integrable distribution, $\dim TM^\omega = \operatorname{codim} M$

$\rightsquigarrow$ characteristic foliation $\mathcal{F}$ on M

Ex: $M = W$, $\mathcal{F}$ = points on W

M Lagrangian $\quad \mathcal{F} = L$ foliation with one leaf

M hypersurface (e.g. $H^{-1}(c)$) $\quad \mathcal{F}$ = orbits of X_H

coisotropic intersection

$\varphi(M) \cap M \neq \emptyset$ under some additional conditions [V.G., Kerman]

Leaf-wise intersections

$\varphi(M) \cap_{leaf} M \neq \emptyset \;:\Leftrightarrow\; \exists$ leaf F of $\mathcal{F}$ s.t. $\varphi(F) \cap F \neq \emptyset$

- $M = W$: $\varphi(M) \cap_{leaf} M \neq \emptyset \Leftrightarrow \varphi$ has fixed point
- M = hypersurface
- $M = L$ $\quad \varphi(L) \cap_{leaf} L = \varphi(L) \cap L$

infinitesimally, $\varphi(M) \cap_{leaf} M \leftrightarrow \operatorname{Crit}(H|_M)$

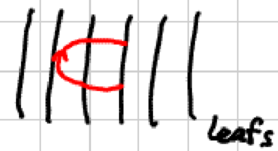
outside connections :

→ geometric mechanics : $\mu : \overset{\circlearrowright G}{W} \to \mathfrak{g}^*$ moment map

$M = \mu^{-1}(0)$ coisotropic , $\mathcal{F}$ = orbits of G

H conserved by G want to find "chords" of φ from leaf to same leaf

$\varphi(M) \cap_{leaf} M = \mathrm{Fix}(\tilde{\varphi} \circlearrowright W/\!/G)$



→ mirror symmetry [Kapustin-Orlov]

A-branes are coisotropic submanifolds with some additional structure

what are morphisms? maybe $Mor(M, \varphi(M)) = \varphi(M) \cap_{leaf} M$

Results

Thm [Moser, Banyaga 1978]

$\varphi \overset{C^1}{\approx} id \Rightarrow \varphi(M) \cap_{leaf} M \neq \emptyset$

approximate Thm : $\varphi \overset{C^0}{\approx} id \Rightarrow \varphi(M) \cap_{leaf} M \neq \emptyset$

- $X_H \overset{C^0}{\approx} 0$ ⇝ OK
- $\varphi_H^t \overset{C^0}{\approx} id$ ⇝ not sure

<u>Thm</u> [Gurel] $\exists$ hypersurface $M \subset \mathbb{R}^{2n}$, $H_i \xrightarrow{C^0} 0$ supported in fixed compact set

s.t. $\varphi_{H_i}(M) \cap_{\text{leaf}} M = \emptyset$

<u>Thm</u> (Gurel) · W exact, $\omega = d\lambda$

· M has "restricted contact type" $\left(\begin{array}{l}\text{If } M \text{ is hypersurface} \\ \lambda|_M \text{ contact}\end{array}\right)$

$\exists C : \varphi_H(M) \cap M \neq \emptyset$ provided $\|H\| < C$

<u>The right question</u>: Assume φ_H is supported in a small nbhd of M

$\overset{?}{\Rightarrow} \varphi(M) \cap_{\text{leaf}} M \neq \emptyset$

<u>Applications</u>:

→ geometric mechanics : small support sufficient

→ Arnold conjecture $\psi \circlearrowright V$ $\#\mathrm{Fix}(\psi) = ?$

Assume $[\sigma] \in H^2(V; \mathbb{Z})$ $\rightsquigarrow$ prequantize

φ = lift of ψ to the prequantized circle bundle M

$\varphi(M) \cap_{\text{leaf}} M \longleftrightarrow \mathrm{Fix}\ \psi$