$$\mathcal{M}_{weak}(L) \xrightarrow{\mathfrak{PO}} \Lambda_{0,nov}$$

$$m_0^b(1) = \sum_{k=0}^{\infty} m_k(\underbrace{b, \dots, b}_{k}) = m(e^b)$$

$$\mathfrak{PO}(b) = \mathfrak{PO}(b) \cdot e_{unit}$$

$b \in \mathcal{M}_{weak}(L)$

Note: $m_0^b(1) = c\, e \Rightarrow m_1^b \circ m_1^b = 0$

$$\mathfrak{PO}^{u_0}(x_1, \cdots, x_n) = \sum_k m_k(\underbrace{b, \cdots, b}_{k}) \cap [L(u_0)]$$

$$b = \sum_{i=1}^{n} x_i \mathbf{e}_i$$

$$\left.\frac{\partial \mathfrak{PO}^{u_0}}{\partial x_i}\right|_b = \sum m_k(\underbrace{b, \cdots, b}_{\ell}, \mathbf{e}_i, \underbrace{b, \cdots, b}_{k-\ell-1}) \cap [L(u_0)]$$

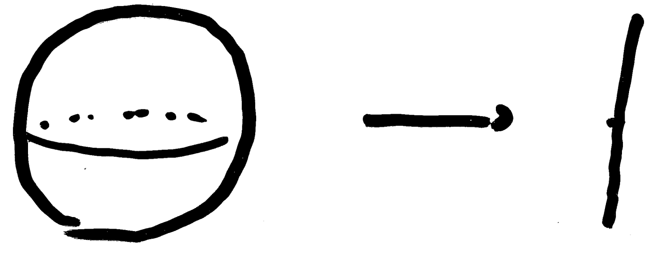
$$= m_1^b(\mathbf{e}_i) \cap [L(u_0)]$$

b "critical point of $\mathfrak{PO}^{u_0}$" $\Rightarrow$ $m_1^b(\mathbf{e}_i) = 0$

- m_1^b compatible with ring str on $H^*(L(u_0) : \Lambda) \cong T^n$

$\Bigg\} \Rightarrow m_1^b = 0$

displacable

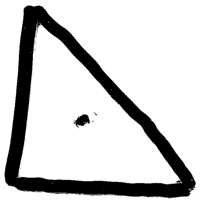


not displacable

2 sol's

$\mathbb{C}P^2 \longrightarrow$

Clifford torus

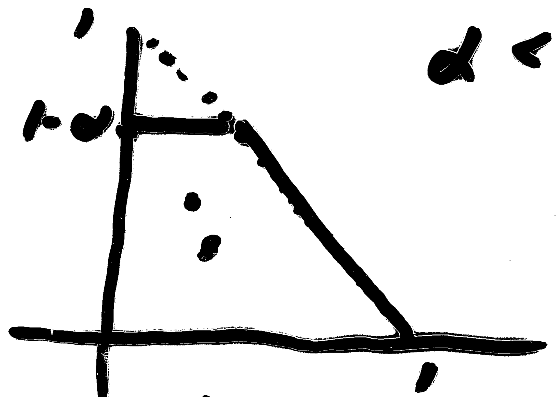


barycenter

3 sol's

$\frac{1}{3} \leq \alpha < 1$

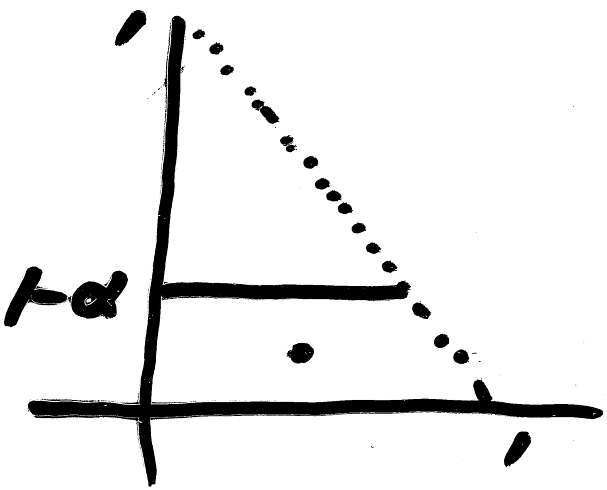
1-pt blow up of $\mathbb{C}P^2 \longrightarrow$



$\alpha < \frac{1}{3}$

$(\frac{1}{3}, \frac{1}{3})$ 3 sol's

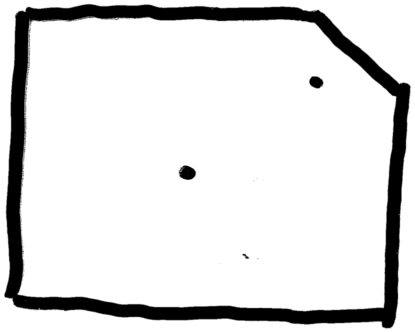
$(\alpha, 1-2\alpha)$ 1 sol



$(\frac{1+\alpha}{4}, \frac{1-\alpha}{2})$

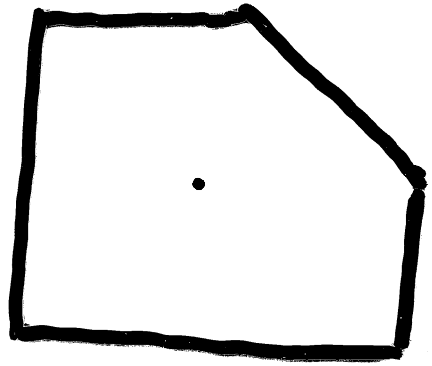
4 sol's

2 pt blow-up of $\mathbb{CP}^2$: I



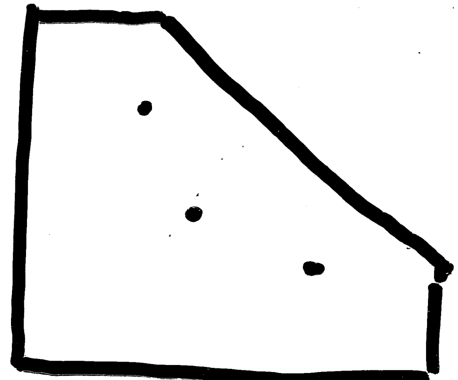
$(0,0)$ 4
(α,α) 1

$\alpha > 0$



$(0,0)$ 5

$\alpha = 0$

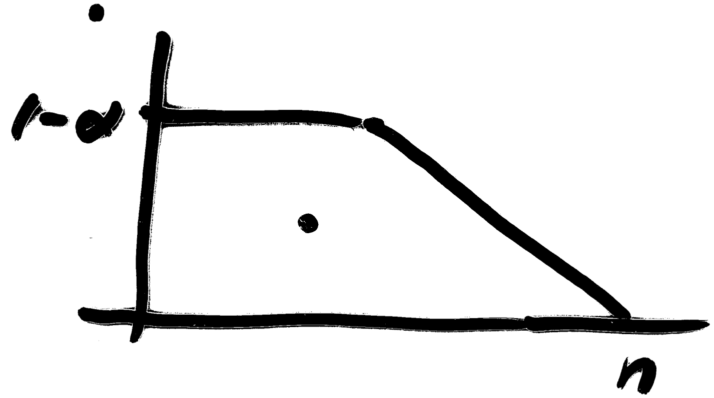


$(\frac{\alpha}{3}, \frac{\alpha}{3})$ 3
$(\alpha+1, \alpha)$ 1
$(\alpha, \alpha+1)$ 1

$\alpha < 0$

$$\left\{ (u_1, u_2) \in \mathbb{R}^2 \;\middle|\; \begin{array}{l} -1 \leq u_1 \leq 1 \\ -1 \leq u_2 \leq 1 \end{array} \quad u_1 + u_2 \leq 1 + \alpha \right\}$$

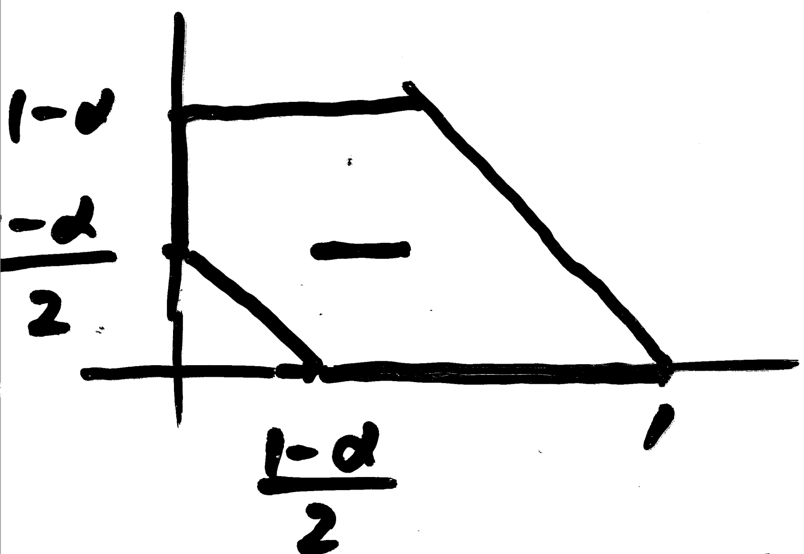
F_n



$\left(\frac{n(\alpha+1)}{4}, \frac{1-\alpha}{2} \right) \in \Delta$ 4 sol's

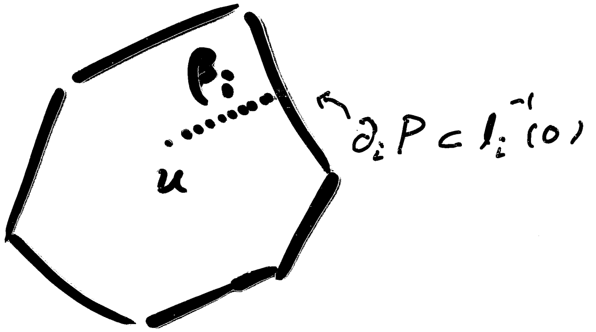
$\left(-\frac{n\alpha}{n-2}, \frac{n-2+2\alpha}{n-2} \right) \notin \Delta$ 1 sol

2 pt blow up of $\mathbb{CP}^2$: II



$\alpha > \frac{1}{3}$

$\left[\frac{1-\alpha}{2}, \frac{1+\alpha}{4} \right] \times \left\{ \frac{1-\alpha}{2} \right\}$



Δ

$$\Delta = \{ u \in t^* \mid l_i(u) \geq 0,\ i = 1, \dots, m \}$$

$$v_i = \nabla l_i \in t$$

$$\int_{\beta_i} \omega = 2\pi\, l_i(u) \qquad \text{Maslov} \quad \mu_{L(u)}(\beta_i) = 2$$

$t^* \cong \mathbb{R}^n$ $\quad u \quad (u_1, \dots, u_n)$

$t \cong \mathbb{R}^n$ $\quad v_i \quad (v_{i,1}, \dots, v_{i,n})$

$$\langle v_i, x \rangle = \sum_j v_{i,j} x_j$$

$$e^{\langle v_i, x\rangle} = e^{v_{i,1} x_1} \cdots e^{v_{i,n} x_n}$$

$$y_1 := e^{x_1}, \dots, y_n = e^{x_n}$$

$$\Rightarrow e^{\langle v_i, x\rangle} = y_1^{v_{i,1}} \cdots y_n^{v_{i,n}} =: y^{v_i}$$

$$\mathfrak{PO}_0^u(x, u) = \sum_{i=1}^m e^{\langle v_i, x\rangle} T^{l_i(u)}$$

leading term potential function

$$= \sum_{i=1}^m \underbrace{y^{v_i} T^{l_i(u)}}_{\overset{!!}{=}\ \bar{z}_i(u)}$$

$$y_j \left(\frac{\partial}{\partial y_j} y^{v_i} \right) = v_{i,j}\, y^{v_i}$$

$$\Rightarrow y_j \frac{\partial}{\partial y_j} \mathfrak{PO}_0^u = \sum v_{i,j}\, y^{v_i} T^{l_i(u)}$$

$$= \sum v_{i,j}\, \bar{z}_i(u)$$

$$\mathfrak{PO}^u = \mathfrak{PO}_0^u + \text{"higher order terms"}$$

X toric mfd

$$\psi_u : QH^{\omega}(X;\Lambda) \longrightarrow Jac(\mathfrak{PO}_0^u)$$

Batyrev quantum cohom.

$$\frac{\Lambda[z_1,\dots,z_m]}{P(X)+SR_\omega(X)} = \frac{\Lambda[y_1,\dots,y_n,y_1^{-1},\dots,y_n^{-1}]}{\left\langle \frac{\partial \mathfrak{PO}_0^u}{\partial y_i} \right\rangle}$$

$P(X)$: linear relation ideal

$SR_\omega(X)$: quantum Stanley-Reisner ideal

$$z_i \longmapsto \bar{z}_i(u) = y^{v_i} T^{\ell_i(u)}$$

$$P(X) \longrightarrow \left\langle \frac{\partial \mathfrak{PO}_0}{\partial y_i} \right\rangle$$

$$c_1(X) = \sum_{i=1}^{m} z_i \longmapsto \mathfrak{PO}_0^u$$

<u>Fano</u>

$$QH(X;\Lambda) \cong Jac(\mathfrak{PO}^u) \qquad \mathfrak{PO}^u = \mathfrak{PO}_0^u \quad (\text{Fano})$$

$$y_i(u) \in QH^{\omega}(X,\Lambda) \xrightarrow{\varphi} \Lambda^{\mathbb{C}}$$

$$\cong$$

$$Jac(\mathfrak{PO}_0^u)$$

$$\|$$

$$y_i \in \frac{\Lambda[y_1, \cdots y_n, y_1^{-1}, \cdots y_n^{-1}]}{\left\langle \frac{\partial \mathfrak{PO}_0^u}{\partial y_i} \right\rangle}$$

<u>Claim</u>

$\exists_1\, u \in t^*$ s.t.

$\mathfrak{v}_T(\varphi(y_i(u)) = 0 \qquad i = 1, \cdots, n$

$x_i := \log y_i(u) \in \Lambda_0$

$b := \sum x_i e_i$

$$\varphi \in Spec(QH^{\omega}(X,\Lambda))(\Lambda^{\mathbb{C}}) \longmapsto (u, b)$$

$$u \in t^*, \quad b \in \mathcal{M}_{weak}(L(u))$$