Cornell Topology Festival 2026
Panel Discussion

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This is a report on the panel discussion of the 61st Topology Festival at Cornell University, which ran from May 1 to May 3, 2026. Each of the speakers was given 5 minutes to outline either a particularly interesting recent result, or an open problem in the field. Summaries of their presentations follow.

Compactification of Teichmüller Space via Projective Filling Currents Space

Christopher Leininger, Rice University

Bonanhon described Thurston compactification of the Teichmüller space in different terms, through the following theorem.

Theorem (Bonanhon, 1988)**.** *The Thurston compactification of the Teichmüller space $\mathcal{T}(S_g)$ can be embedded into the space of currents $Cur(S_g)$, specifically, into the projective space of currents $\mathbb{P}Cur(S_g)$.*

In the spirit of this theorem, Walsh gave an alternative description of the Thurston compactification as the horofunction compactification with respect to the Thurston's asymmetric Lipschitz metric. Further advancing the above two results, Meenakshy Joythis and Dídac Martínez-Granado proved the following.

Theorem (Joythis–Martínez-Granado, 2026)**.** *The space $\mathbb{P}Cur_{fill}(S_g)$ of projective filling currents admits an extension of Thurston's asymmetric Lipschitz metric. Moreover, the horofunction compactification in Walsh's result is exactly all space of currents $\mathbb{R}Cur(S_g)$.*

On the Existence of High Dimensional Complex Structures

Corey Bregman, Tufts University

Definition. Given a $2n$-manifold M, an **almost complex structure** is map $J : TM \to TM$ such that $J^2 = \mathrm{Id}_{TM}$.

Question (Yau)**.** Do there exist almost complex (but not complex) manifolds with dim ≥ 6?

Question. Does $\mathbb{S}^6$ admit a complex structure?

Given M_1, M_2 closed, orientable manifolds, the fact that both M_i are parallelizable implies that $M_1 \times M_2$ is almost complex. So we have the following question:

Question. For which M_1, M_2 is $M_1 \times M_2$ complex?

For many Seifert fibered manifolds, we have positive examples for the above question.

Fact. *Given M^3 hyperbolic, then $M^3 \times SO(3)$ is complex, because if we set $\Gamma = \pi_1(M^3)$, then $PSL_2(\mathbb{C})/\Gamma = F_n^{\perp}(M^3)$.*

Complete curves in strata of differentials

Dawei Chen, Boston College

Here, we consider complete curves over $\mathbb{C}$. Let $\mathcal{M}_g$ be the moduli space of genus g algebraic curves, and suppose that $g \geq 3$.

Fact. *There exist complete curves in $\mathcal{M}_g$.*

Let $\mathcal{H}(\mu)$ be the moduli space of differentials of on genus g surfaces whose singularities have orders corresponding to the partition μ of $2g-2$. Let $\mathbb{P}\mathcal{H}(\mu) = \mathcal{H}(\mu)/\mathbb{C}^*$.

Question. Is there a complete curve in $\mathbb{P}\mathcal{H}(\mu)$?

There does not exist a complete curve in $\mathcal{H}(\mu)$ when μ is holomorphic type, i.e., it is a positive partition (Gendron). There is not a complete curve in $\mathcal{H}(\mu)$ or $\mathbb{P}\mathcal{H}(\mu)$ when μ is meromorphic type i.e. it is a partition with some negative entries (Chen).

The following question remains open.

Question. Is there a complete curve in a component of $\mathbb{P}\mathcal{H}(\mu)$ when μ is of holomorphic type, $g \geq 6$, and the component is non-hyperelliptic?

Knot invariants under connected sum

Samantha Allen, Duquesne University

Question (Kirby, 1997)**.** Is the unknotting number additive under connected sum? That is, is $u(K\#J) = u(K) + u(J)$ for all knots K and J?

Theorem (Brittenham–Hermiller, 2025)**.** *The (2,7)-torus knot T and its mirror $\overline{T}$ satisfy $u(T\#\overline{T}) \leq 5$, while $u(T) = u(\overline{T}) = 3$.*

The knot T in the theorem has only 14 crossings!

Question. Do we always have $u(K_1\# \cdots \#K_n) \geq n$ for any n knots $K_1, \ldots, K_n$?

This question has an affirmative answer for $n = 2$, due to Schnarlmann.

These questions are still open for the crossing number c in place of the unknotting number u. A recent result in this direction is as follows.

Theorem (Lackenby, 2009)**.** *For any n knots $K_1, \dots, K_n$,*

$$\frac{c(K_1) + \cdots + c(K_n)}{152} \leq c(K_1 \# \cdots \# K_n).$$

The stable homology of Hurwitz spaces

Kirsten Wickelgren, Duke University

Landesman and Levy have recently made great strides in a program started by Ellenberg–Venkatesh–Westerland, which seeks to extract number theoretic results from the stable homology of Hurwitz spaces. The Hurwitz space $\mathrm{Hur}_n^d(\mathbb{C})$ is the space of degree d covers branched at n points of $\mathbb{C}$.

The bridge is the Grothendieck–Lefschetz trace formula, which for a smooth scheme X relates the count of $\mathbb{F}_q$-points of X to the topology—in particular, the cohomology—of the complex points $X(\mathbb{C})$. For the Hurwitz space Hur_n^d, point counts over $\mathbb{F}_q$ are related to several important counting problems in number theory, including

- Malle's conjecture on the number of Galois extensions with a given Galois group,
- distributions of Selmer groups of elliptic curves,
- the Cohen–Lenstra heuristics on distributions of class groups,

over function fields $\mathbb{F}_q(t)$.

The Kervaire invariant in dimension 126

Thomas Brazelton, Harvard University

The cup product gives rise to a symmetric bilinear form on the middle cohomology group $H^{2n}(M)$ of a connected oriented $4n$-manifold M, whose signature is the so-called *signature* of M.

If M is instead a $(4n+2)$-dimensional manifold, one can still make sense of this form on $H^{2k+1}(M; \mathbb{Z}/2)$, now with mod 2 coefficients. If M is furthermore framed, this form admits a quadratic refinement, whose Arf invariant is the so-called *Kervaire invariant* of M, an element of $\mathbb{Z}/2$. The Kervaire invariant is 0 if and only if M can be surgically converted into a sphere.

In his resolution of the Hopf invariant problem, Adams studied the elements

$$h_j \in \mathrm{Ext}_{\mathcal{A}}^{1,2^j}(\mathbb{F}_2, \mathbb{F}_2) \Longrightarrow \pi_{2^j-1}(\mathbb{S}_2^\wedge)$$

in the E^2-page of the so-called Adams spectral sequence, which converges to the stable homotopy groups of spheres (at the prime 2). Here, $\mathcal{A}$ is the *Steenrod algebra*, the graded ring of cohomology operations with mod 2 coefficients.

Theorem (Browder, 1969). *There exists an n-manifold with Kervaire invariant 1 if and only if*

1. $n = 2^{j+1} - 2$, *and*
2. *the class h_j^2 survives the Adams spectral sequence.*

The classes h_j^2 were known to survive for $j \leq 5$ before 2009, when Hill, Hopkins, and Ravenel showed that h_j^2 does not survive for $j \geq 7$. This left open the fate of h_6^2.

Theorem (Lin–Wang–Xu, 2025). *h_6^2 survives the Adams spectral sequence.*

As a consequence, there exist 126-manifolds with Kervaire invariant 1. One of the major technical advancements behind this result is Weinan Lin's implementation of noncommutative Gröbner bases for the Steenrod algebra $\mathcal{A}$.

Curve counts over finite fields

Rohini Ramadas, Emory University

Theorem (Canning–Lawson–Payne–Willwacher, 2024). *The number of genus g curves over the field $\mathbb{F}_q$ is a polynomial function in q if and only if $g \leq 8$.*

The Flat Closing Problem

Jingyin Huang, Ohio State University

Let M be a closed, negatively curved manifold ($K < 0$).

Theorem (Pressman). *There is no copy of $\mathbb{Z}^2$ in $\pi_1(M)$.*

Relaxing the condition to non-positive curvature ($K \leq 0$) leads to:

Theorem (Flat Torus Theorem). *If $\mathbb{Z}^2 \leq \pi_1(M)$, then there exists a flat in the universal cover $\widetilde{M}$.*

A *flat* is an isometric embedding $\mathbb{E}^2 \hookrightarrow \widetilde{M}$. In the 1970s, the following converse was proposed:

Question (Flat Closing). Suppose there exists a 2-flat in $\widetilde{M}$. Can we find a $\mathbb{Z}^2 \leq \pi_1(M)$?

Theorem (Martelli, 2025). *No.*

Martelli produced an example of a compact space with flats, but this example is not a manifold. The question remains open for the specific case of manifolds.